ANALİZ I
UYGULAMA

1.Hafta

-İnfimum ve supremum
-Yığılma Noktası
-Sonlu Seri Toplamı
-Sonlu Çarpım

Örnek: Aşağıdaki kümelerin varsa sup ve inf bulunuz.

a) $A=\left\{\frac{1}{r} : r\in\mathbb{Q} \text{ ve } r>0\right\} \Rightarrow \sup A = 0^{\notin A}$, $\inf A$ yok

b) $B=\{x : 1<x<2,\ x\in\mathbb{R}\} \Rightarrow \sup B = 2^{\notin B}$, $\inf B = 1^{\notin B}$

c) $C=\left\{\frac{n^2+1}{n} : n\in\mathbb{N}\right\} = \left\{2, \frac{5}{2}, \frac{10}{3}, \ldots n+\frac{1}{n}, \ldots\right\}$ sup C yok $\inf C = 2^{\in C}$

Örnek: $A=\left\{\frac{1}{n} : n\in\mathbb{N}\right\}$ yığılma noktaları? 0.

Örnek: $B=[0,1]$ yığılma noktaları kümesi: $[0,1]$

Örnek: $C=[-1,0)\cup(0,1]$ y.n.k: $[-1,1]$

Örnek: $\mathbb{Q}$ nun yığ.n.k: $\mathbb{R}$

$\mathbb{R}/\mathbb{Q}$ nın y.n.k: $\mathbb{R}$

$\mathbb{N}$ nin y.n.k: $\emptyset$ yoktur

Seri Toplam Özellikleri

a_k, b_k birer dizi ve $\alpha\in\mathbb{R}$ olsun.

i) $$\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

ii) $$\sum_{k=1}^{n}\alpha a_k = \alpha\sum_{k=1}^{n} a_k$$

iii) $$\sum_{k=1}^{n} a_n = \sum_{k=1}^{j} a_n + \sum_{k=j}^{n} a_n$$

Uygulama

1) $$\sum_{k=1}^{50} k = 1+2+\cdots+50 = \frac{50.51}{2} = 1275$$

2) $$\sum_{k=1}^{10} k^2 = 1^2+2^2+\cdots+10^2 = \frac{10.(10+1)(2.10+1)}{6} = \frac{\cancel{10}^{5}.11.\cancel{21}^{7}}{\cancel{6}} = 385$$

3) $$\sum_{k=1}^{25} (-1)^k k = \underbrace{\underbrace{(-1)+2}_{1}+\underbrace{(-3)+4}_{1}+\cdots+\underbrace{(-23)+24}_{1}}_{12}+(-25)$$
$$12 + (-25) = -13$$

4) $$\sum_{k=1}^{10} k^3 = \left(\frac{10.11}{2}\right)^2 = 55^2$$

5) $$\sum_{k=1}^{80} = \left(\sqrt{k+1}-\sqrt{k}\right) = (\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{81}-\sqrt{80}) = -1+\sqrt{81}$$

6) $$\sum_{k=1}^{10} \frac{1}{k.(k+1)} = ? \qquad \frac{1}{k.(k+1)} = \frac{A}{k}+\frac{B}{k+1} = \frac{Ak+A+Bk}{k.(k+1)} \Rightarrow \begin{matrix} A+B=0 \\ A=1 \end{matrix} \Rightarrow B=-1$$

$$\hookrightarrow = \sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+1}\right) = \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{10}-\frac{1}{11}\right) = 1-\frac{1}{11}$$

7) $$\sum_{k=1}^{120} \frac{1}{k\sqrt{k+1}+(k+1)\sqrt{k}}$$

$$\frac{1}{\sqrt{k}\sqrt{k+1}\left(\sqrt{k}+\sqrt{k+1}\right)} = \frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k}\sqrt{k+1}\,(k+1-k)} = \frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k}\sqrt{k+1}} = \frac{\sqrt{k+1}}{\sqrt{k}\sqrt{k+1}} - \frac{\sqrt{k}}{\sqrt{k}\sqrt{k+1}}$$
$$= \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}$$

$$\Rightarrow \sum_{k=1}^{120}\left(\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}\right) = \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{120}-\frac{1}{121}\right)$$

$$= 1-\frac{1}{121}$$

8) $\sum_{k=1}^{40} 2^k = \frac{1-2^{41}}{1-2} - 1 = 2^{41} - 2$

$S_n = 1 + r + r^2 + \cdots + r^n$

$rS_n = r + r^2 + r^3 + \cdots + r^{n+1}$

$S_n(1-r) = 1 - r^{n+1} \Rightarrow S_n = \frac{1-r^{n+1}}{1-r}$

$S_n - 1$

9) $\sum_{k=1}^{40} k \cdot 2^k = 39 \cdot 2^{41} + 2$

$K_n = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n 2^n$

$2K_n = 1 \cdot 2^2 + 2 \cdot 2^3 + 3 \cdot 2^4 + \cdots + n 2^{n+1}$

$K_n - 2K_n = 2 + 2^2 + 2^3 + \cdots + 2^n - n 2^{n+1}$

$S_n = 1 + r + r^2 + \cdots + r^n \Rightarrow S_n = \frac{1-r^{n+1}}{1-r}$

$K_n - 2K_n = -1 + \underbrace{1 + 2^2 + 2^3 + \cdots + 2^n}_{\frac{1-2^{n+1}}{1-2}} - n 2^{n+1}$

$-K_n = -1 + 2^{n+1} - 1 - n 2^{n+1}$

$\boxed{K_n = (n-1) 2^{n+1} + 2}$

10) $\sum_{k=1}^{25} \frac{k}{(k+1)!} = \left(\frac{1}{1} - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \cdots + \left(\frac{1}{25!} - \frac{1}{26!}\right) = 1 - \frac{1}{26!}$

$\frac{k+1-1}{(k+1)!} = \frac{1}{k!} - \frac{1}{(k+1)!}$

11) $\prod_{k=1}^{10} k = 1.2.\cdots.10 = 10!$

12) $\prod_{k=3}^{50} (k-17) = -16.(-15).\cdots.0\cdots22.23 = 0$

13) $\prod_{k=1}^{50} \frac{k}{k+1} = \frac{1}{2}.\cancel{\frac{2}{3}}.\cancel{\frac{3}{4}}.\cdots.\cancel{\frac{49}{50}}.\frac{50}{51} = \frac{1}{51}$

14) $\prod_{k=1}^{51} (51k-k^2) = (51-1)(102-4)\cdots(\underbrace{51.51-51^2}_{0}) = 0$

15) $\prod_{k=2}^{20} \left(1-\frac{1}{k^2}\right) = \prod_{k=2}^{20} \frac{(k+1)(k-1)}{k^2} = \prod_{k=2}^{20} \frac{k+1}{k} \cdot \prod_{k=2}^{20} \frac{k-1}{k}$

$= \frac{3}{2}.\cancel{\frac{4}{3}}.\cancel{\frac{5}{4}}\cdots\frac{21}{20} \cdot \frac{1}{2}.\cancel{\frac{2}{3}}.\cancel{\frac{3}{4}}\cdots\frac{19}{20}$

$= \frac{21}{40}$

16) $\sum_{k=1}^{25} k.k! = \sum (k+1-1)k! = \sum \left[(k+1)!-k!\right]$

$= (2!-1)+(3!-2!)+(4!-3!)+\cdots+(26!-25!)$

$= 26!-1$

17) $\sum_{k=-13}^{13} \frac{1}{5^k+1}$

k=1'den k=13'e kadar $a_n = \frac{1}{5^n+1}$

ise k=-1'den k=-13'kadar $a_{-n} = \frac{1}{5^{-n}+1}$

$a_0 = \frac{1}{5^0+1} = \boxed{\frac{1}{2}}$

$\frac{1}{5^n+1} + \frac{1}{5^{-n}+1} = \frac{1}{5^n+1} + \frac{1}{\frac{1}{5^n}+1} \Rightarrow \frac{1+5^n}{5^n+1} = 1$

(5^n)

Her çift için 1 geliyor O zaman

$\sum_{k=-13}^{13} \frac{1}{5^k+1} = \frac{1}{2} + \sum_{k=1}^{13} 1 = \frac{1}{2} + 13$