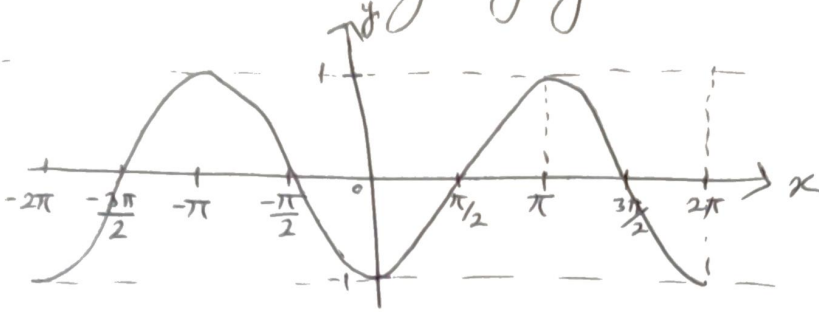


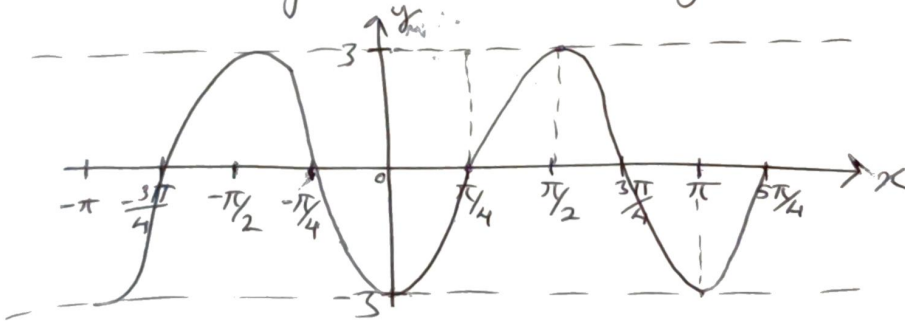
Analiz I - 3. Uygulama

1) $y = \sin(x - \frac{\pi}{2})$ nin grafiğini çiz.

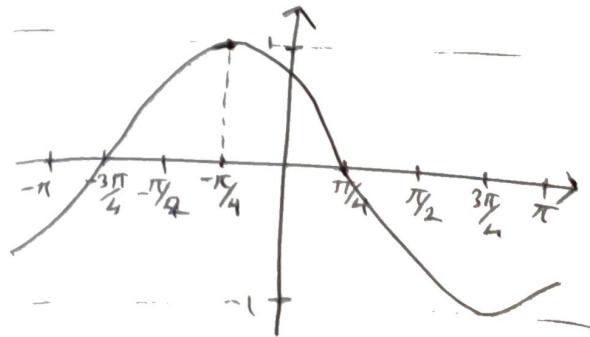
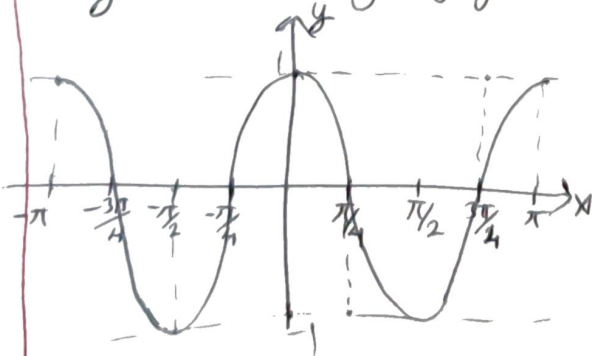


2) $y = 3\sin(2(x - \frac{\pi}{4}))$ grafiğini çiz.

~~uç değerleri~~ Görüntü kümesi: $[-3, 3]$
x-ekseni boyunca $\frac{\pi}{4}$ sağa ötelenmiş.
Periyodu 2π den π ye dönüşmüş.



3) a) $y = \cos(2x)$ grafiğini çiz b) $y = \cos(x + \frac{\pi}{4})$ grafiğini çiz

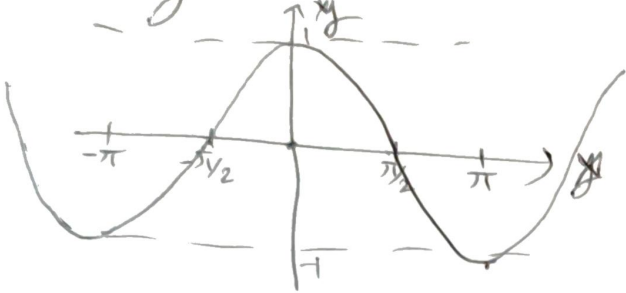


c) $y = 2\cos(2(x + \frac{\pi}{4}))$ ödev.

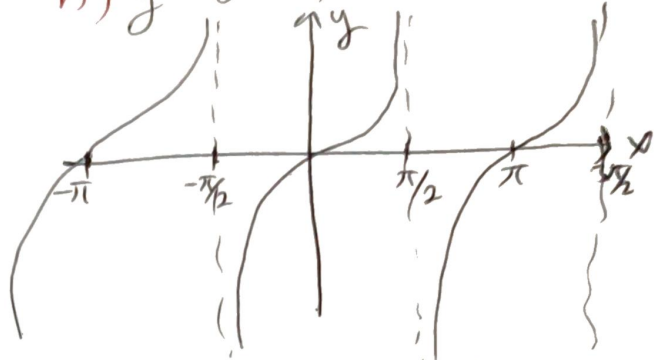
Bunu yapma →

3) Aşağıdakilerin grafiklerini çiziniz

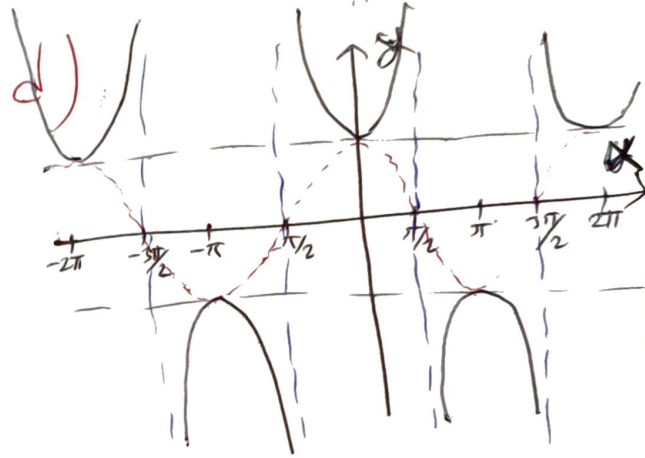
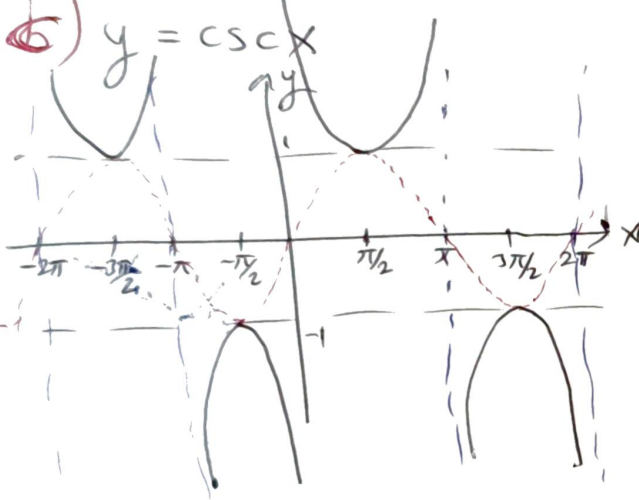
a) $y = \cos x$



b) $y = \tan x$



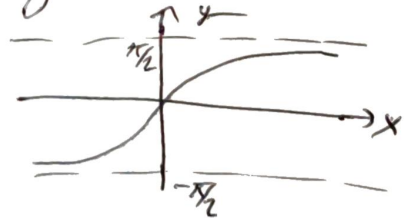
c) $y = \csc x$



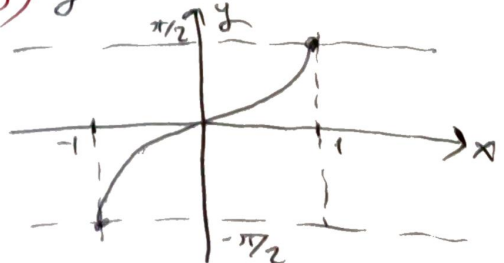
e) ~~$y = \arccos x$~~
 $y = \cot x$



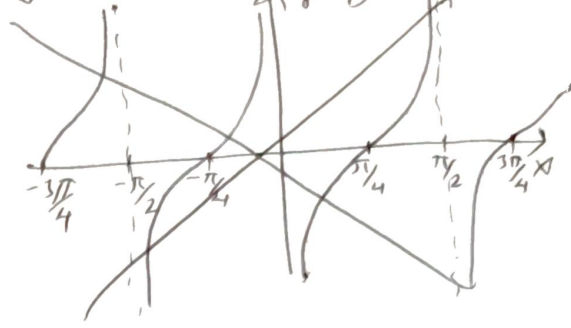
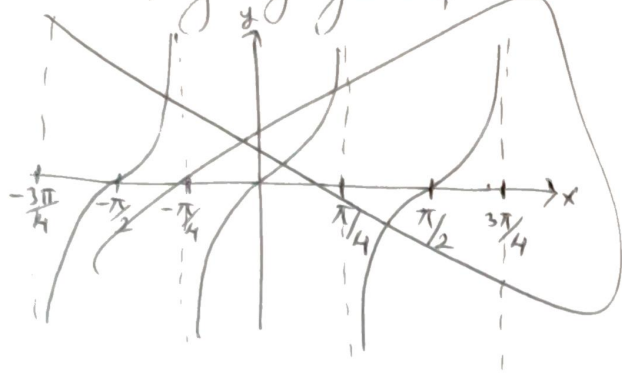
f) $y = \arctan x$



g) $y = \arcsin x$



4) a) $y = \tan(2x)$ grafiğini çiziniz b) $y = \tan(x - \pi/4)$ graf. çiz.



5) $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ eşitliğini kullanarak $\cos \frac{\pi}{12}$ yi bulun

$$\begin{aligned}\cos \frac{\pi}{12} &= \cos \left(\frac{\pi}{3} - \frac{\pi}{4} \right) = \cos \frac{\pi}{3} \cdot \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

6) $\sin u = -\frac{3}{5}$, $\cos u = \frac{12}{13}$ ve $\pi < u < \frac{3\pi}{2}$, $\frac{3\pi}{2} < u < 2\pi$

o.ä. $\sin(u+v)$ yi bulunuz.

$$\sin^2 u + \cos^2 u = 1 \text{ old}$$

$$\cos^2 u = 1 - \sin^2 u$$

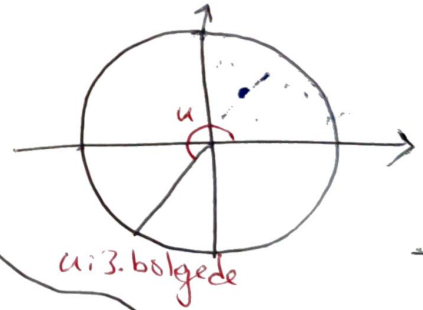
$$\cos u = \pm \sqrt{1 - \sin^2 u}$$

$$\Rightarrow \cos u = -\sqrt{1 - \sin^2 u} \quad (u: 3. \text{ bölgede old})$$

$$= -\sqrt{1 - \left(-\frac{3}{5}\right)^2}$$

$$= -\frac{4}{5}$$

$$\boxed{\cos u = -\frac{4}{5}}$$



$\cos u$ ve $\sin u$ yi bulunuyoruz.

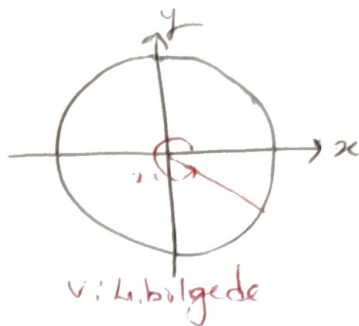
$$\sin u = \pm \sqrt{1 - \cos^2 u}$$

$$\sin u = -\sqrt{1 - \cos^2 u} \quad (u, \text{IV. halbkreis})$$

$$= -\sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$= -\frac{5}{13}$$

$$\boxed{\sin u = -\frac{5}{13}}$$



$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$= \left(-\frac{3}{5}\right)\left(\frac{12}{25}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) = -\frac{16}{65}$$

7) $\frac{\sin(x-y)}{\cos x \cos y} = \tan x - \tan y$ old. gös.

$$\frac{\sin(x-y)}{\cos x \cos y} = \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y}$$

$$= \frac{\cancel{\sin x} \cos y}{\cos x \cancel{\cos y}} - \frac{\cos x \cancel{\sin y}}{\cancel{\cos x} \cos y} \quad \begin{matrix} \text{tan x} & \text{tan y} \end{matrix}$$

$$= \tan x - \tan y$$

8) $\tan(\pi - x) = -\tan x$ old. gös.

$$\tan(\pi - x) = \frac{\tan \pi - \tan x}{1 + \tan \pi \tan x}$$

$$= \frac{0 - \tan x}{1 + 0 \cdot \tan x}$$

$$= -\tan x$$

9) ~~cos~~ $\cos \theta = -\frac{3}{5}$ ve θ , II. bölgede
ise aşağıdakileri hesaplayınız.

Hatırlatma

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$

a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

Önce ne $\tan \theta$ 'yı bulalım.

$$\begin{aligned}\sin \theta &= \sqrt{1 - \cos^2 \theta} \\ &= \sqrt{1 - \left(-\frac{3}{5}\right)^2} = \frac{4}{5} \Rightarrow \tan \theta = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}\end{aligned}$$

$$\begin{aligned}\text{a) } \sin 2\theta &= 2 \sin \theta \cdot \cos \theta \\ &= 2 \cdot \frac{4}{5} \cdot \left(-\frac{3}{5}\right) = -\frac{24}{25}\end{aligned}$$

$$\begin{aligned}\text{b) } \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = -\frac{7}{25}\end{aligned}$$

$$\begin{aligned}\text{c) } \tan 2\theta &= \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta} = \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2 \cdot \left(-\frac{4}{3}\right)}{1 - \left(-\frac{4}{3}\right)^2} = \frac{24}{7}\end{aligned}$$

10) a) $\cos\left(\frac{7\pi}{12}\right)$ b) $\sin\left(\frac{7\pi}{12}\right)$ c) $\cos\left(\frac{7\pi}{6}\right)$ d) $\sin\left(\frac{7\pi}{6}\right)$

$$\begin{aligned}\text{a) } \cos \frac{7\pi}{12} &= \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}\end{aligned}$$

$$\text{b) } \sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4} = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\text{c) } \cos \frac{7\pi}{6} = \cos\left(2 \cdot \frac{7\pi}{12}\right) = \cos^2\left(\frac{7\pi}{12}\right) - \sin^2\left(\frac{7\pi}{12}\right)$$

$$\text{d) } \sin \frac{7\pi}{6} = \sin\left(2 \cdot \frac{7\pi}{12}\right) = 2 \sin \frac{7\pi}{12} \cdot \cos \frac{7\pi}{12}$$

11) En geniş tanım kümesini bulun.

a) $\log_x 5 \Rightarrow x \neq 1, x > 0$ b) $\log_5 x \Rightarrow x > 0$

$x=1$ olsa $1^y=5$ o.e. y olmalı.

12) Aşağıdakileri en sade şekilde yazınız.

a) $\log 3x - \log 4y = \log\left(\frac{3x}{4y}\right)$

b) $2\ln x + \frac{1}{2}\ln(x^2+1) = \ln x^2 + \ln(x^2+1)^{1/2} = \ln(x^2\sqrt{x^2+1})$

c) $2\log_2 5 + \log_2 9 - \log_2 75 = \log_2 25 + \log_2 9 - \log_2 75$
 $= \log_2\left(\frac{25 \cdot 9}{75}\right) = \log_2 3$

13) $4 + 3\log_2 x = 1$ denkleminin çözüm kümesini bul

$$\left. \begin{aligned} 3\log_2 x &= -3 \Rightarrow \log_2 x = -1 \\ &\Rightarrow x = 2^{-1} \\ &\quad \boxed{x = \frac{1}{2}} \end{aligned} \right\} \begin{array}{l} \text{Kontrol edelim} \\ 4 + 3\log_2 2^{-1} = 1 \\ 4 - 3 = 1 \quad \checkmark \end{array}$$

14) $\log_2(x-3) + \log_2(x-4) = 1$

$$\begin{aligned} \log_2((x-3)(x-4)) &= 1 \\ \Rightarrow (x-3)(x-4) &= 2^1 \\ x^2 - 7x + 10 &= 0 \\ x &= 2 \text{ ve } x = 5 \\ \text{Kontrol edelim} \end{aligned}$$

$x=2$ için
 $\log_2 1 + \log_2^{-2} = 1$
 ~~$\log_2 1$~~ sağlanmaz
Çünkü $x < 0$ iken
tanımlı değil.

$x=5$ için
 $\log_2 2 + \log_2 1 = 1$
 $1 + 0 = 1$
Sağlandı
 $x \in \{5\}$