ANALİZ III
UYGULAMA

1.HAFTA

-Kısmi Toplamlar Serisi
-Seri Toplamı
-Oran Testi

Analiz III – Uygulama 1. Hafta

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1) $(a_n)_n$ yakınsak o.ü. $\sum_{n=1}^{\infty}(a_n - a_{n+1})$ serisi de yakınsaktır.

Gösteriniz ve toplamını bulunuz.

$$(S_n) = \sum_{i=1}^{n}(a_i - a_{i+1}) = (a_1 - \cancel{a_2}) + (\cancel{a_2} - \cancel{a_3}) + \dots + (\cancel{a_{n-1}} - \cancel{a_n}) + (\cancel{a_n} - a_{n+1})$$

$$= a_1 - a_{n+1}$$

$$\Rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty}(a_1 - a_{n+1}) = a_1 - \lim_{n\to\infty} a_{n+1} = \sum_{n=1}^{\infty}(a_n - a_{n+1})$$

2) $\sum_{n=1}^{\infty}\left(\sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}\right) = ?$

$$S_n = \sum_{k=1}^{n}\left(\sqrt{k+2} - 2\sqrt{k+1} + \sqrt{k}\right)$$

$$= \sum_{k=1}^{n}\left(\sqrt{k+2} - \sqrt{k+1}\right) + \sum_{k=1}^{n}\left(\sqrt{k} - \sqrt{k+1}\right)$$

$$= (\cancel{\sqrt{3}} - \sqrt{2}) + (\cancel{\sqrt{4}} - \cancel{\sqrt{3}}) + \dots + (\sqrt{n+2} - \cancel{\sqrt{n+1}}) + (\sqrt{1} - \cancel{\sqrt{2}}) + (\cancel{\sqrt{2}} - \cancel{\sqrt{3}}) + \dots + (\cancel{\sqrt{n}} - \sqrt{n+1})$$

$$= -\sqrt{2} + \sqrt{n+2} + 1 - \sqrt{n+1}$$

$$\Rightarrow \lim_{n\to\infty} S_n = \lim\left(1 - \sqrt{2} + \sqrt{n+2} - \sqrt{n+1}\right) = \lim_{n\to\infty}\left(1 - \sqrt{2} + \frac{(\sqrt{n+2} - \sqrt{n+1})(\sqrt{n+2} + \sqrt{n+1})}{(\sqrt{n+2} + \sqrt{n+1})}\right)$$

$$= \lim_{n\to\infty}\left(1 - \sqrt{2} + \underbrace{\frac{n+2-(n+1)}{(\sqrt{n+2} + \sqrt{n+1})}}_{n\to\infty \; 0}\right)$$

$$\boxed{= 1 - \sqrt{2}}$$

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3) $$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1 \text{ old. gös.}$$

$$S_n = \sum_{k=1}^{n} \frac{k}{(k+1)!} = \sum_{k=1}^{n} \frac{k \cdot k!}{(k+1)!\, k!}$$

$$= \sum_{k=1}^{n} \frac{1}{k!} - \frac{1}{(k+1)!}$$

$$= 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3!}} + \cancel{\frac{1}{3!}} - \cancel{\frac{1}{4!}} + \dots + \cancel{\frac{1}{n!}} - \frac{1}{(n+1)!}$$

$$= 1 - \frac{1}{(n+1)!}$$

$$\lim S_n = \lim_{n\to\infty} \left(1 - \frac{1}{(n+1)!}\right) = 1$$

4) $$\sum_{n=1}^{\infty} \left(\sqrt{k+1} - \sqrt{k}\right) = +\infty \text{ old. gös.}$$

$$S_n = \sum_{k=1}^{n} \left(\sqrt{k+1} - \sqrt{k}\right) = \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \dots + \sqrt{n+1} - \sqrt{n}$$

$$= \sqrt{n+1} - 1$$

$$\lim S_n = \lim_{n\to\infty} \sqrt{n+1} = \infty$$

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5) $\sum_{n=1}^{\infty} \frac{4}{(4k-3)(4k+1)} = 1$ old. gös.

$$S_n = \sum_{k=1}^{n} \frac{4}{(4k-3)(4k+1)}$$

$$\frac{4}{(4k-3)(4k+1)} = \frac{A}{(4k-3)} + \frac{B}{(4k+1)}$$

$\Rightarrow$ $4kA + A + 4kB - 3B = 4$

$A - 3B = 4$ $\quad 4A + 4B = 0$

$A = -B$

$\Rightarrow -B - 3B = 4 \Rightarrow B = -1$
$A = 1$

$$S_n = \sum_{k=1}^{n} \left(\frac{1}{4k-3} + \frac{-1}{4k+1} \right)$$

$$= 1 - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \frac{1}{9} + \frac{1}{9} - \frac{1}{13} + \cdots + \frac{1}{4n-3} - \frac{1}{4n+1}$$

$$= 1 - \frac{1}{4n+1}$$

$$\lim S_n = \lim_{n\to\infty} \left(1 - \frac{1}{4n+1}\right) = 1$$

6) $\sum_{k=1}^{\infty} k r^{k-1} = \frac{1}{(1-r)^2}$ $(|r|<1)$ old. gös.

$$S_n = \sum_{k=1}^{n} k r^{k-1} = 1 + 2r + 3r^2 + 4r^3 + \cdots + n r^{n-1}$$

$$r S_n = r + 2r^2 + 3r^3 + \cdots + n r^n$$

$$S_n - r S_n = \underbrace{1 + r + r^2 + \cdots + r^{n-1}}_{\frac{1-r^n}{1-r}} + n r^n \Rightarrow S_n(1-r) = \frac{1-r^n}{1-r} + n r^n$$

$$\Rightarrow S_n = \frac{1-r^n}{(1-r)^2} + n r^n \Rightarrow \lim S_n = \lim_{n\to\infty} \frac{1-r^n}{(1-r)^2} + \underbrace{\lim_{n\to\infty} n r^n}_{0} = \frac{1}{(1-r)^2}$$

7) 6. Sorudan yararlanarak

a) $\sum_{k=1}^{\infty} \frac{k}{2^k}$ b) $\sum_{k=1}^{\infty} (-1)^k \frac{k}{3^k}$ c) $\sum_{k=1}^{\infty} \frac{2k+1}{5^k}$

serilerinin toplamını bulunuz.

a) $\sum k \left(\frac{1}{2}\right)^k = \frac{1}{2} \sum k \left(\frac{1}{2}\right)^{k-1}$

$$= \frac{1}{2} \left(\frac{1}{\left(1-\frac{1}{2}\right)^2} \right)$$

b ve c ödev.

8) $\sum_{k=1}^{\infty} \frac{1}{k^2+k+1}$ serisinin yakınsaklığını inceleyiniz

$\frac{1}{k^2+k+1} < \frac{1}{k^2}$ ve $\sum_{k=1}^{\infty} \frac{1}{k^2}$ yakınsak old

karşılaştırma testinden yak-dır.

9) $\sum_{k=1}^{\infty} \frac{k}{e^k}$ inceleyin

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(k+1)}{e^{k+1}} \cdot \frac{e^k}{k} = \lim \frac{k+1}{e \cdot k} = \frac{1}{e} < 1$$

old. yakınsaktır.

(Hatırlatma $\sum a_k$ var

$\lim \frac{a_{k+1}}{a_k} = r \Rightarrow$ $r<1 \Rightarrow$ yak. $r>1 \Rightarrow$ ırak. $r=1 \Rightarrow$ yanıt vermez.)

10) $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ inceleyiniz.

$$\lim_{n\to\infty} \frac{((n+1)!)^3}{(3n+3)!} \cdot \frac{(3n)!}{(n!)^3} = \lim_{n\to\infty} \frac{(n!(n+1))^3 \cdot (3n)!}{(n!)^3 \cdot (3n+3)(3n+2)(3n+1)[(3n)]!}$$

$$= \lim_{n\to\infty} \frac{(n+1)^3}{(3n+3)(3n+2)(3n+1)} = \frac{1}{27} < 1$$

old. yak.

11) ~~lim~~ $\sum_{k=1}^{\infty} \frac{e^k}{k}$ inceleyiniz

$\frac{1}{k} < \frac{e^k}{k}$ old. ıraksaktır.

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KAYNAKLAR

[1] Balcı M., & Aral A.,(2001). Matematik Analiz 2. Balcı Yayınları. (Vol. 2)
[2] Adams, R. A., & Essex, C. (1999). Calculus: a complete course (Vol. 4). Boston: Addison-Wesley.